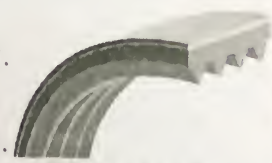
Flexible mechanical elements (belts, chains, ropes) are used in conveying systems and to transmit power over long distances (*instead of using shafts and gears*).

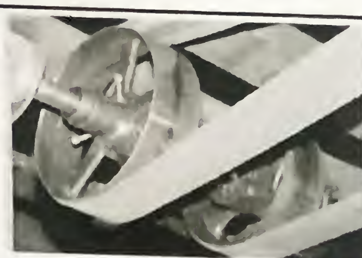
- The use of flexible elements simplifies the design and reduces cost.
- Also, since these elements are elastic and usually long, they play a role in absorbing shock loads and reducing vibrations.
- Disadvantage, they have shorter life than gears, shafts, etc.

Belts

- There are four basic types of belts (*Table 17-1*):
  - Flat belts ~ *crowned pulleys.*
  - Round belts ~ *grooved pulleys.*
  - V-belts ~ *grooved pulleys.*
  - Timing belts ~ *toothed pulleys.*

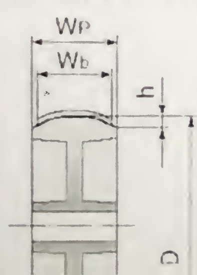


Ribbed belt



Flat belt

- Characteristics of belt drives:
  - Pulley axis must be separated by certain minimum distance.
  - Can be used for long center distances.
  - Except for timing belts, there is some slipping between belt and pulley, thus angular velocity ratio is not constant or equal to the ratio of pulley diameters. ≠ d Pully ratio
  - A tension pulley can be used to maintain tension in the belt.



Belts continued

- There are <u>two</u> main configurations for belt drives; <u>open</u> and <u>crossed</u> (*Fig 17-1*) where the direction of rotation will be reversed for the crossed belt drive.
- The figure shows <u>reversing</u> and <u>non-reversing</u> belt drives, always there is one *loose side* depending on the <u>driver pulley</u> and the <u>direction of rotation</u>.

Figure 17-1

Flat-belt geometry. (a) Open belt. (b) Crossed belt

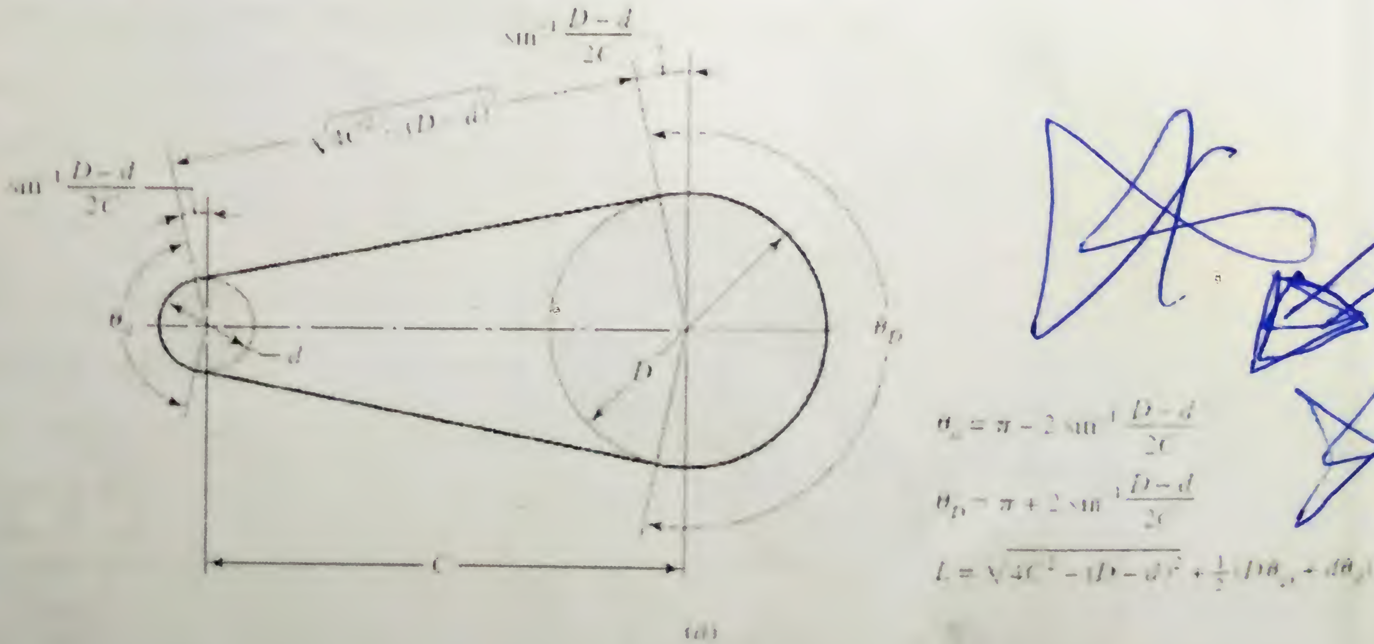


Figure 17-1

Flat-belt geometry. (a) Open belt. (b) Crossed belt.

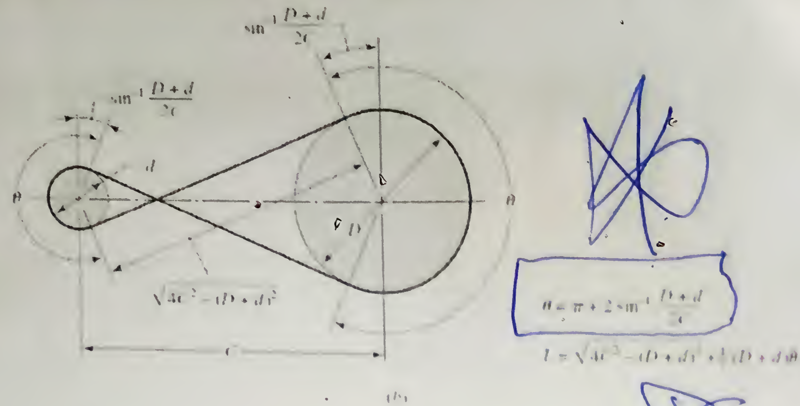


Fig. 17–2-a: shows the loose and tight sides of the belt.

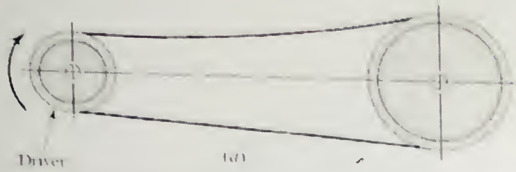
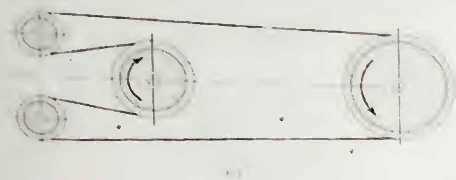


Fig. 17–2-c: shows reversing open-belt drive



Belt continued

Fig. (17-3) shows flat belt drive for out of-plane pulleys.

Fig. (17-4) shows how clutching action can be obtained by shifting the belt from loose to a tight pulley.

Fig. (17-5) shows two types of variable-speed belt drives.

Figure 17-3

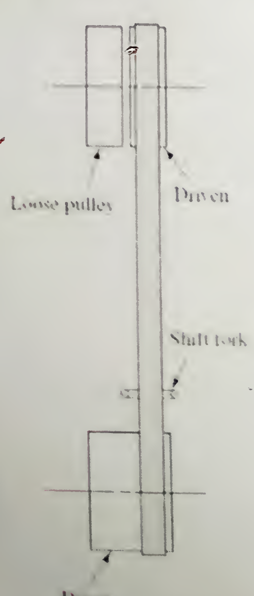


Figure 17-4

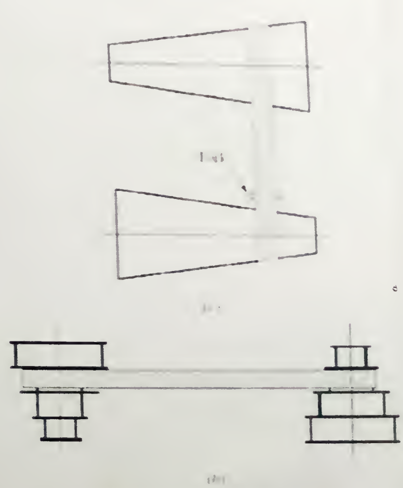


Figure 17-5

Flat belt drivers produce very little noise and they absorb more vibration from the system than V-belts.

Also, flat belts drives have high efficiency of about *98 %* (*same as for gears*) compared to *70-96 %* for V-belts.

- For open belt drives, the contact angles

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} \qquad (17\text{-}1)$$

where: D : diameter of larger pulley

d : diameter of smaller pulley

C : centers distance

Open

Flat and Round Belt Drive continued

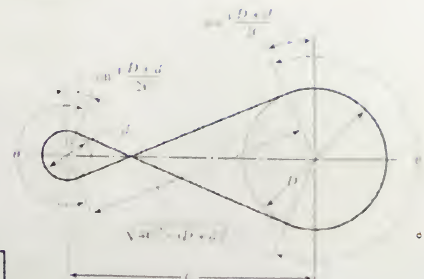
- And the length of the belt is:

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d) \qquad (17\text{-}2)$$

- For crossed belt drives, the contact angle is the same for both pulleys:

oss

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C} \qquad (17\text{-}3)$$



- And the belt length is:

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{1}{2}(D+d)\theta \qquad (17\text{-}4)$$

- Tight side tension:

$$\boxed{\begin{aligned} F_1 &= F_i + F_c + \Delta F' \\ &= F_i + F_c + T/D \end{aligned}}$$

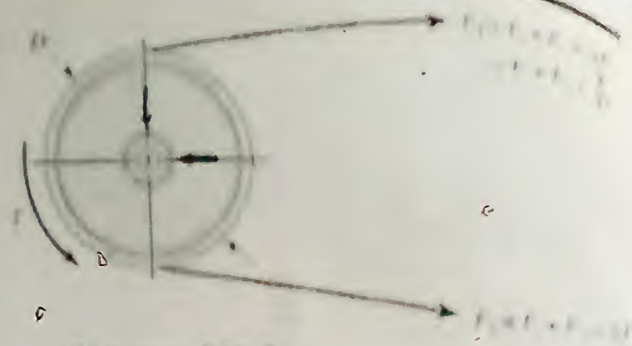


Figure 17-7

Forces and torques on a pulley

- Loose side tension:

$$\boxed{\begin{aligned} F_2 &= F_i + F_c - \Delta F' \\ &= F_i + F_c - T/D \end{aligned}}$$

where F_i : initial tension, F_c : hoop tension due to centrifugal force, and $\Delta F'$: tension due to transmitted torque.

- The total transmitted force is the difference between F_1 & F_2

$$\boxed{F_1 - F_2 = \frac{2T}{D}} \quad \boxed{\text{.............. (*)}}$$

$\Delta F = \frac{2T}{D}$

Force Analysis continued

- The centrifugal tension F_c can be found as:

$$F_c = mr^2\omega^2$$

where ω: is the angular velocity, & m: is the mass per unit length.

It also can be written as:

- The belting equation relates the possible belt tension values with the coefficient of friction and it is defined as:

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{f\phi} \qquad (17\text{-}7)$$

Note that ϕ is the smallest value of the contact angle

where f: coefficient of friction, ϕ: contact angle.

- By dividing equation 1 by equation (*), and using the last equation, we can find the relation between F_i and T as given below:

$$\boxed{F_i = \frac{T}{D}\frac{e^{f\phi}+1}{e^{f\phi}-1}} \qquad (17\text{-}9).$$

Minimum value of F_i needed to transmit a certain value of torque without slipping

- This equation shows that if F_i is zero; then T is zero (*i.e. there is no transmitted torque*).

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

when $F_c = 0 \Rightarrow F_i = \frac{F_1 + F_2}{2}$

when $F_i = 0 \Rightarrow \boxed{F_c = \frac{F_1 + F_2}{2}}$

Force Analysis continued

Flat and Round-Belt Materials

Properties of Some Flat- and Round-Belt Materials. (Diameter = d, thickness = t, width = w)

Material	Specification	Size, in	Minimum Pulley Diameter, in	Allowable Tension per Unit Width at 600 ft/min, lbf/in	Specific Weight, lbf/in³	Coefficient of Friction
Leather	1 ply	$t = \frac{11}{64}$	3	30	0.035–0.045	0.4
		$t = \frac{13}{64}$	$3\frac{1}{2}$	33	0.035–0.045	0.4
	2 ply	$t = \frac{18}{64}$	$4\frac{1}{2}$	41	0.035–0.045	0.4
		$t = \frac{20}{64}$	6[a]	50	0.035–0.045	0.4
		$t = \frac{23}{64}$	9[a]	60	0.035–0.045	0.4
Polyamide[b]	F–0[c]	$t = 0.03$	0.60	10	0.035	0.5
	F–1[c]	$t = 0.05$	1.0	35	0.035	0.5
	F–2[c]	$t = 0.07$	2.4	60	0.051	0.5
	A–2[c]	$t = 0.11$	2.4	60	0.037	0.8
	A–3[c]	$t = 0.13$	4.3	100	0.042	0.8
	A–4[c]	$t = 0.20$	9.5	175	0.039	0.8
	A–5[c]	$t = 0.25$	13.5	275	0.039	0.8
Urethane[d]	$w = 0.50$	$t = 0.062$	See	5.2[e]	0.038–0.045	0.7
	$w = 0.75$	$t = 0.078$	Table	9.8[e]	0.038–0.045	0.7
	$w = 1.25$	$t = 0.090$	17–3	18.9[e]	0.038–0.045	0.7
	Round	$d = \frac{1}{4}$	See	8.3[e]	0.038–0.045	0.7
		$d = \frac{3}{8}$	Table	18.6[e]	0.038–0.045	0.7
		$d = \frac{1}{2}$	17–3	33.0[e]	0.038–0.045	0.7
		$d = \frac{3}{4}$		74.3[e]	0.038–0.045	0.7

Table 17-2

Belt Style	Belt Size, in	Ratio of Pulley Speed to Belt Length, rev/(ft · min) Up to 250	250 to 499	500 to 1000
Flat	0.50 × 0.062	0.38	0.44	0.50
	0.75 × 0.078	0.50	0.63	0.75
	1.25 × 0.090	0.50	0.63	0.75
Round	$\frac{1}{4}$	1.50	1.75	2.00
	$\frac{3}{8}$	2.25	2.62	3.00
	$\frac{1}{2}$	3.00	3.50	4.00
	$\frac{3}{4}$	5.00	6.00	7.00

Table 17–3

Force Analysis continued

*Average values of C_p for the given ranges were approximated from curves in the *Habasit Engineering Manual*, Habasit Belting, Inc., Chamblee (Atlanta), Ga.

Velocity Factor

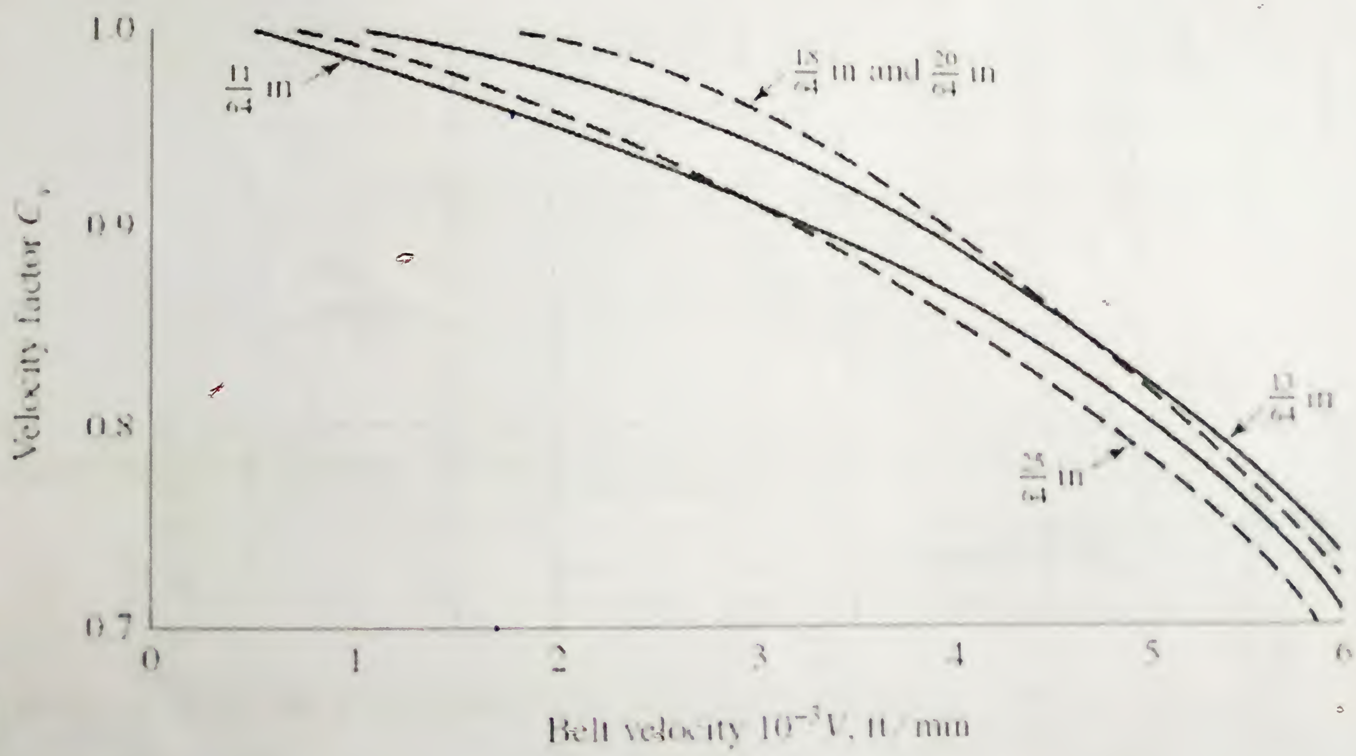


Figure 17–9

Velocity correction factor C_v for leather belts for various thicknesses. (Data source: Machinery's Handbook, 20th ed., Industrial Press, New York, 1976, p. 1047.)

- The transmitted horsepower can be found as:

$$\boxed{H = (F_1 - F_2)\, V / 33000}$$

- However, when designing a design factor n_d needs to be included to account for unquantifiable effects. Also another correction factor K_s, is included to account for load deviations from the nominal value (*i.e., over loads*).

- Thus the design horsepower is:

$$\boxed{H_d = H_{nom} K_s n_d}$$ ⟹ Used when designing a belt drive

Note: K_s can be obtained from Table 17-15

Suggested Steps for Analyzing Flat Belts

6. Find $\boxed{F_2}$

$$F_2 = (F_1)_a - ((F_1)_a - F_2)$$

Note that F_2 must be larger than zero

7. From $(F_1)_a$, F_2 & F_c find $\boxed{F_i}$.

8. Check if the friction of the belt material is sufficient to transmit the torque

$$f' < f$$

where $$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c}$$

Minimum friction needed to transmit the load without slipping

9. Find the factor of safety $\boxed{n_{fs}} = H_a / H_{nom} K_s$

or Hd mostly

Used in analysis, in which H_a is the allowable power calculated based on $(F_1)_a$

* H_{nom} is also known as the power capacity of the belt drive.

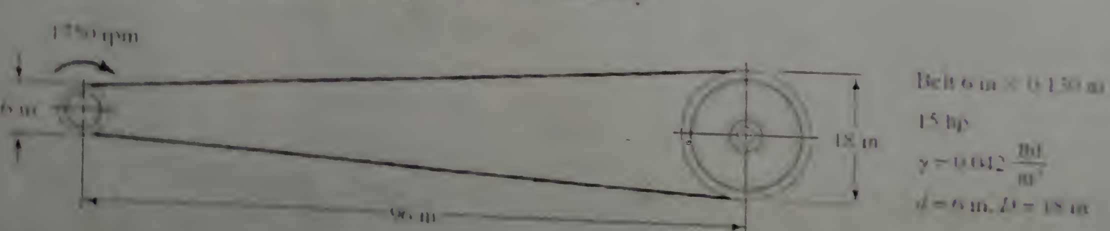
Example 17-1

A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(a) Estimate the centrifugal tension F_c and the torque T.

(b) Estimate the allowable F_1, F_2, F_i and allowable power H_a.

(c) Estimate the factor of safety. Is it satisfactory?



Solution (*a*) Eq. (17–1):

$$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{18-6}{2(8)12}\right] = 3.0165 \text{ rad}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$ width x thick times

Table 17–2:

$$w = 12\gamma bt = 12(0.042)6(0.130) = 0.393 \text{ lbf/ft}$$

Answer Eq. (*e*):

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.393}{32.17}\left(\frac{2749}{60}\right)^2 = 25.6 \text{ lbf}$$

$$T = \frac{63\,025 H_{nom} K_s n_d}{n} = \frac{63\,025(15)1.25(1.1)}{1750}$$

Answer

$$= 742.8 \text{ lbf} \cdot \text{in}$$

(*b*) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (*h*), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

Example 17-1 continued

Answer The combination $(F_1)_a$, F_2, and F_c will transmit the design power of $15(1.25)(1.1) = 20.6$ hp and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{420 - 25.6}{172.4 - 25.6} = 0.328$$

From Table 17–2, $f = 0.8$. Since $f' < f$, that is, $0.328 < 0.80$, there is no danger of slipping.

(c)

Answer
$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1 \qquad \text{(as expected)}$$

Answer The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 20.6 hp.

Example 17-1 (Alternative solution)

EXAMPLE 17–1 A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$.
The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10.

Estimate the factor of safety. Is it satisfactory?

Figure 17–10

Solution

Eq. (17–1):
$$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{18 - 6}{2(8)12}\right] = 3.0165 \text{ rad}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$

Table 17–2: $w = 12\gamma bt = 12(0.042)6(0.130) = 0.393$ lbf/ft

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.393}{32.17}\left(\frac{2749}{60}\right)^2 = 25.6 \text{ lbf}$$

From Table 17–2 $F_a = 100$ lbf. For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

$$(F_1)_a = bF_aC_pC_v = 6(100)0.70(1) = 420 \text{ lbf}$$

We know that $\frac{F_1 - F_c}{F_2 - F_c} = e^{f\phi}$, but the limiting value of F_1 is $(F_1)_a$, so we can write:

$$\frac{(F_1)_a - F_c}{F_2 - F_c} = e^{f\phi} \Rightarrow \frac{420 - 25.6}{F_2 - 25.6} = 11.17$$, from which $F_2 = 60.9$ lb

Now, the allowable power $H_a = ((F_1)_a - F_2)V/33000$

or $H_a = (420 - 60.9)\,2749/33000 = 29.9$ hp

Now, the factor of safety is obtained from $n_{fs} = H_a/(H_{nom}K_s)$

That is $n_{fs} = 29.9/(15)(1.25) = 1.59$

So the belt drive is satisfactory (safe) since the factor of safety is greater than 1 .

Dip of Flat Belts

The dip d of flat belt is related to the initial tension by the following relation :

F_i

$$d = \frac{12L^2w}{8F_i} = \frac{3L^2w}{2F_i} \qquad (17\text{–}13)$$

where d = dip, in

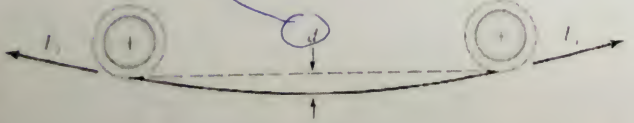
L = center-to-center distance, ft

w = weight per foot of the belt, lbf/ft

F_i = initial tension, lbf

In Ex. 17–1 the dip corresponding to a 270.6-lb initial tension is

$$d = \frac{3(8^2)0.393}{2(270.6)} = 0.14 \text{ in}$$



A decision set for a flat belt can be

- Function: power, speed, durability, reduction, service factor, C
- Design factor: n_d
- Initial tension maintenance
- Belt material
- Drive geometry, d, D
- Belt thickness: t
- Belt width: b

Depending on the problem, some or all of the last four could be design variables. Belt cross-sectional area is really the design decision, but available belt thicknesses and widths are discrete choices. Available dimensions are found in suppliers' catalogs.

Summary Note

One can show that the following relations are correct:

$$w = 12\gamma bt = (12\gamma t)b = a_1 b.$$

$$(F_1)_a = F_a b C_p C_v = (F_a C_p C_v)b = a_0 b$$

$$F_c = \frac{wV^2}{g} = \frac{a_1 b}{32.174}\left(\frac{V}{60}\right)^2 = a_2 b$$

$$(F_1)_a - F_2 = 2T/d = 33\,000 H_d/V = 33\,000 H_{nom} K_s n_d/V$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = a_0 b - 2T/d$$

$$f\phi = \ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \ln\frac{(a_0 - a_2)b}{(a_0 - a_2)b - 2T/d}$$

$$b_{min} = \frac{1}{a_0 - a_2}\,\frac{33\,000 H_d}{V}\,\frac{\exp(f\phi)}{\exp(f\phi) - 1}$$

or

$$1 \quad \left[\frac{33\,000 H_d \exp(f\phi)}{\ldots}\right]$$

EXAMPLE 17-2 Design a flat-belt drive to connect horizontal shafts on 16-ft centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 60 hp under very light shock.

Solution

- Function: $H_{nom} = 60$ hp, 860 rev/min, 2.25:1 ratio, $K_s = 1.15$, $C = 16$ ft
- Design factor: $n_d = 1.05$
- Initial tension maintenance: catenary
- Belt material: polyamide
- Drive geometry, d, D
- Belt thickness: t
- Belt width: b

The last four could be design variables. Let's make a few more a priori decisions.

Decision $d = 16$ in, $D = 2.25d = 2.25(16) = 36$ in.

Example 17-2 continued

Estimate centrifugal tension F_c in terms of belt width b:

$$w = 12\gamma bt = 12(0.042)b(0.13) = 0.0655b \text{ lbf/ft}$$

$$V = \pi dn/12 = \pi(16)860/12 = 3602 \text{ ft/min}$$

Eq. (e):
$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.0655b}{32.17}\left(\frac{3602}{60}\right)^2 = 7.34b \text{ lbf} \qquad (2)$$

For design conditions, that is, at H_d power level, using Eq. (h) gives

$$(F_1)_a - F_2 = 2T/d = 2(5310)/16 = 664 \text{ lbf} \qquad (3)$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 94.0b - 664 \text{ lbf} \qquad (4)$$

Using Eq. (i) gives

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{94.0b + 94.0b - 664}{2} - 7.34b = 86.7b - 332 \text{ lbf} \qquad (5)$$

Place friction development at its highest level, using Eq. (17–7):

$$f\phi = \ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \ln\frac{94.0b - 7.34b}{94.0b - 664 - 7.34b} = \ln\frac{86.7b}{86.7b - 664}$$

Solving the preceding equation for belt width b at which friction is fully developed gives

$$b = \frac{664}{86.7}\frac{\exp(f\phi)}{\exp(f\phi) - 1} = \frac{664}{86.7}\frac{11.38}{11.38 - 1} = 8.40 \text{ in}$$ *(Min. b for no slipping)*

A belt width greater than 8.40 in will develop friction less than $f = 0.80$. The manufacturer's data indicate that the next available larger width is 10 in.

Shigley's Mechanical Engineering Design

Example 17-2 *continued*

Example 17-2 continued

Decision Use 10-in-wide belt.

It follows that for a 10-in-wide belt

Eq. (2): $$F_i = 7.34(10) = 73.4 \text{ lbf}$$

Eq. (1): $$(F_1)_a = 94(10) = 940 \text{ lbf}$$

Eq. (4): $$F_2 = 94(10) - 664 = 276 \text{ lbf}$$

Eq. (5): $$F_i = 86.7(10) - 332 = 535 \text{ lbf}$$

The transmitted power, from Eq. (3), is

$$H_t = \frac{[(F_1)_a - F_2]V}{33\,000} = \frac{664(3602)}{33\,000} = 72.5 \text{ hp}$$

and the level of friction development f', from Eq. (17-7) is

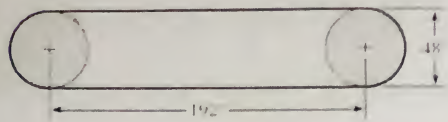
$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.037} \ln \frac{940 - 73.4}{276 - 73.4} = 0.479$$

which is less than $f = 0.8$, and thus is satisfactory. Had a 9-in belt width been available, the analysis would show $(F_1)_a = 846$ lbf, $F_2 = 182$ lbf, $F_i = 448$ lbf, and $f' = 0.63$. With a figure of merit available reflecting cost, thicker belts (A-4 or A-5) could be examined to ascertain which of the satisfactory alternatives is best. From Eq. (17-13) the catenary dip is

$$d = \frac{3L^2 w}{2F_i} = \frac{3(15^2)(0.0655)(10)}{2(535)} = 0.413 \text{ in}$$

A flat-belt drive is to consist of two 4-ft-diameter cast-iron pulleys spaced 16 ft apart. Select a belt type to transmit 60 hp at a pulley speed of 380 rev/min. Use a service factor of 1.1 and a design factor of 1.0.

Solution:



A priori decisions:

- Function: $H_{nom} = 60$ hp, $n = 380$ rev/min, $VR = 1$, $C = 192$ in, $K_s = 1.1$
- Design factor: $n_d = 1$
- Belt material: Polyamide A-3
- Drive geometry: $d = D = 48$ in
- Belt thickness: $t = 0.13$ in

Design variable: Belt width b

Solution of Problem 17-3 continued

Use a method of trials. Initially choose $b = 6$ in

$$V = \frac{\pi d n}{12} = \frac{\pi(48)(380)}{12} = 4775 \text{ ft/min}$$

$$w = 12\gamma b t = 12(0.042)(6)(0.13) = 0.393 \text{ lbf/ft}$$

$$F_c = \frac{wV^2}{g} = \frac{0.393(4775/60)^2}{32.174} = 77.4 \text{ lbf}$$

$$T = 63\,025 H_{nom} K_s n_d / n \implies T = \frac{63\,025(1.1)(1)(60)}{380} = 10\,946 \text{ lbf} \cdot \text{in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(10\,946)}{48} = 456.1 \text{ lbf}$$

$$F_1 = (F_1)_a = b F_a C_p C_v = 6(100)(1)(1) = 600 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 600 - 456.1 = 143.9 \text{ lbf}$$

$$f' = \frac{1}{\phi_d} \ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right) = \frac{1}{\pi} \ln\left(\frac{600 - 77.4}{143.9 - 77.4}\right) = 0.656 < f = 0.8 \text{, so okay}$$

$$L = [4C^2 - (D-d)^2]^{1/2} + \frac{1}{2}(D\theta_D + d\theta_d) \implies L = 534.8 \text{ in}$$

So we choose polyamide A-3 flat belt with $b = 6$ in, $t = 0.13$ in, and L = 534.8 in.

Note: $b_{min} = 5.7$; which makes f' equals $f = 0.8$ and makes F_1 and F_2 min. ; and this improves the life of the belt. But unfortunately, $b = 5.7$ in is not available and the nearest available belt width is $b = 6$ in.

The cross sectional dimensions of V-belts are standardized. Each letter designates a certain cross section (see *Table 17-9*).

- A V-belt can be specified by the cross section letter followed by the *inside* circumference length.
  - ❖ *Table 17-10* gives the standard lengths for V-belts.
  - However, calculations involving the belt length are usually based on *pitch length* for standard belts.
  - ❖ Table 17-11 *gives the quantity to be added to the inside length.*

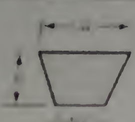
    Example: Pitch length of B75 belt is 75+1.8 = 76.8 mm

- The standard angle for the V-belts cross section is 40°; *however the sheave angle is* slightly smaller causing the belt to wedge itself *inside the sheave to increase* friction.
- The operating speed for V-belts needs to be high *and the recommended speed* range is from *5 to 25 m/s*. Best *performance is obtained at speed of 20 m/s*.

V- Belts continued

Table 17-9

Standard V-Belt Sections



Belt Section	Width *a*, in	Thickness *b*, in	Minimum Sheave Diameter, in	hp Range, One or More Belts
A	$\frac{1}{2}$	$\frac{11}{32}$	3.0	$\frac{1}{4}$–10
B	$\frac{21}{32}$	$\frac{7}{16}$	5.4	1–25
C	$\frac{7}{8}$	$\frac{17}{32}$	9.0	15–100
D	$1\frac{1}{4}$	$\frac{3}{4}$	13.0	50–250
E	$1\frac{1}{2}$	1	21.6	100 and up

L

Table 17-10 Inside Circumferences of Standard V Belts

Section	Circumference, in
A	26, 31, 33, 35, 38, 42, 46, 48, 51, 53, 55, 57, 60, 62, 64, 66, 68, 71, 75, 78, 80, 85, 90, 96, 105, 112, 120, 128
B	35, 38, 42, 46, 48, 51, 53, 55, 57, 60, 62, 64, 65, 66, 68, 71, 75, 78, 79, 81, 83, 85, 90, 93, 97, 100, 103, 105, 112, 120, 128, 131, 136, 144, 158, 173, 180, 195, 210, 240, 270, 300
C	51, 60, 68, 75, 81, 85, 90, 96, 105, 112, 120, 128, 136, 144, 158, 162, 173, 180, 195, 210, 240, 270, 300, 330, 360, 390, 420
D	120, 128, 144, 158, 162, 173, 180, 195, 210, 240, 270, 300, 330, 360, 390, 420, 480, 540, 600, 660
E	180, 195, 210, 240, 270, 300, 330, 360, 390, 420, 480, 540, 600, 660

Table 17-11

Length Conversion Dimensions (Add the Listed Quantity to the Inside Circumference to Obtain the Pitch Length in Inches)

Belt section	A	B	C	D	E
Quantity to be added	1.3	1.8	2.9	3.3	4.5

$B112 \therefore L = 112, \; L_p = L + L_c$

V- Belts continued

- Horsepower:
  - *Table 17-12* gives the horsepower rating for each belt cross-section (according to *sheave pitch diameter and belt speed*).
  - The allowable horsepower per-belt, H_a is found as:

$$\boxed{H_a = K_1 K_2 H_{tab}} \qquad (17\text{-}17)$$

Power that can be transmitted by each belt

from Table 17-12

where,

K_1: contact angle correction factor (*Table 17-13*).

Note: the contact angles for V-belts are found using the same equations used for flat belts.

K_2: belt length correction factor (*Table 17-14*).

V- Belts continued

Table 17-12

Horsepower Ratings of Standard V Belts

Belt Section	Sheave Pitch Diameter, in	Belt Speed, ft/min 1000	2000	3000	4000	5000
A	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
B	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
C	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
D	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
E	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 17-13

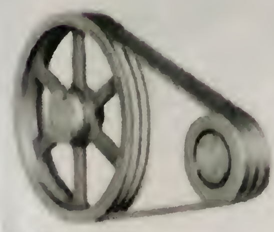
Angle of Contact Correction Factor K_1 for VV* and V-Flat Drives

$\frac{D-d}{C}$	θ, deg	K_1 VV	K_1 V Flat
0.00	180	1.00	0.75
0.10	174.3	0.99	0.76
0.20	166.5	0.97	0.78
0.30	162.7	0.96	0.79
0.40	156.9	0.94	0.80
0.50	151.0	0.93	0.81
0.60	145.1	0.91	0.83
0.70	139.0	0.89	0.84
0.80	132.8	0.87	0.85
0.90	126.5	0.85	0.85
1.00	120.0	0.82	0.82
1.10	113.3	0.80	0.80
1.20	106.3	0.77	0.77
1.30	98.9	0.73	0.73
1.40	91.1	0.70	0.70
1.50	82.8	0.65	0.65

*A curvefit for the VV column in terms of θ is

$K_1 = 0.143\,543 + 0.007\,46\,8\theta - 0.000\,015\,052\,\theta^2$

in the range $90° \le \theta \le 180°$



VV: all pulleys are v-grooved

K1

V-flat drive: is a V-belt drive which uses a flat pulley on one or more shafts.

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D 360

V- Belts continued

Table 17-14

Belt-Length Correction Factor K_2*

K2

Length Factor	Nominal Belt Length, in A Belts	B Belts	C Belts	D Belts	E Belts
0.85	Up to 35	Up to 46	Up to 75	Up to 128	
0.90	38–46	48–60	81–96	144–162	Up to 195
0.95	48–55	62–75	105–120	173–210	210–240
1.00	60–75	78–97	128–158	240	270–300
1.05	78–90	105–120	162–195	270–330	330–390
1.10	96–112	128–144	210–240	360–420	420–480
1.15	120 and up	158–180	270–300	480	540–600
1.20		195 and up	330 and up	540 and up	660

D

V- Belts continued

- The belting equation for V-belts is the same equation used for flat belts. The effective coefficient of friction for *Gates Rubber Company* belts is *0.5123*

Thus,
$$\frac{F_1 - F_c}{F_2 - F_c} = e^{0.5123\phi} \qquad (17\text{–}18)$$

for the V-belt company
$f = \text{constant}$
$= 0.5123$

- Where the centrifugal tension F_c is found as:

$$F_c = K_c \left(\frac{V}{1000} \right)^2 \qquad (17\text{–}21)$$

K_c: accounts for mass of the belt (*Table 17-16*).

Table 17–16

Some V-Belt Parameters*

Belt Section	K_b	K_c
A	220	0.561
B	576	0.965
C	1 600	1.716
D	5 680	3.498
E	10 850	5.041
3V	230	0.425
5V	1098	1.217
8V	4830	3.288

The power that is transmitted per belt is based on $\Delta F = F_1 - F_2$, where

$$\boxed{\Delta F = \frac{63\,025 H_d / N_b}{n(d/2)}} \qquad (17\text{–}22)$$

where n (rpm) and d (in) are for the driver pulley.

From the definition of ΔF, the least tension F_2 is

$$\boxed{F_2 = F_1 - \Delta F} \qquad (17\text{–}24)$$

then from Eq. (17–8) the largest tension F_1 is given by

$$\boxed{F_1 = F_c + (F_1 - F_2)\frac{e^{f\phi}}{e^{f\phi} - 1}} \qquad (17\text{–}23)$$

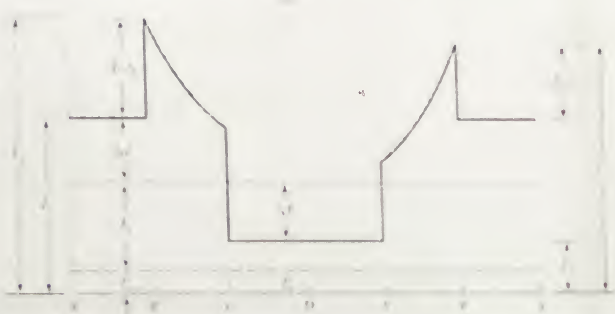
And F_i is found as: $$\boxed{F_i = \frac{F_1 + F_2}{2} - F_c} \qquad (17\text{–}25)$$

The factor of safety is $$\boxed{n_{fs} = \frac{H_a N_b}{H_{nom} K_s}} \qquad (17\text{–}26)$$

$\frac{H_a}{H_{nom}\ K_s}$

V- Belts continued

- In flat-belt force analysis, the tension induced from bending the belt was ignored (*since belt thickness is not that large*), however, in V-belts the effect of flexural stress is more pronounced, and thus it affects the durability (*life*) of the belt. The figure shows the two tension peaks T_1 & T_2 resulting from belt flexure.



  - The values of tension peaks are found as:

$$\boxed{T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d}} \qquad \boxed{T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D}}$$

- The life of V-belts is defined as the number of passes the belt can do (N_p), and it is found as:

$$\boxed{N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}} \qquad (17\text{-}27)$$

❖ where K & b are found from *Table 17-17*

2 Or life time in hours is found as:

$$\boxed{t = \frac{N_P L_p}{720V}} \qquad (17\text{-}28)$$

, where V is in units of ft/min and L_p in inches

Note: K & b values given in *Table 17-17* are valid only for the indicated range. Thus, if N_p is found to be larger than 10^9 it is reported as N_p=10^9 and life time in hours "t" is found using N_p=10^9.

V- Belts continued

Table 17-17

Durability Parameters for Some V-Belt Sections

Belt Section	10^8 to 10^9 Force Peaks K	10^8 to 10^9 Force Peaks b	10^9 to 10^{10} Force Peaks K	10^9 to 10^{10} Force Peaks b	Minimum Sheave Diameter, in
A	674	11.089			3.0
B	1193	10.926			5.0
C	2038	11.173			8.5
D	4208	11.105			13.0
E	6061	11.100			21.6
3V	728	12.464	1062	10.153	2.65
5V	1654	12.593	2394	10.283	7.1
8V	3638	12.629	5253	10.319	12.5

The analysis of a V-belt drive can consist of the following steps:

- Find V, L_p, C, ϕ, and $\exp(0.5123\phi)$
- Find H_d, H_a, and N_b from H_d/H_a and round up
- Find F_c, ΔF, F_1, F_2, and F_i, and n_{fs}
- Find belt life in number of passes, or hours, if possible

N_b t

A 10-hp split-phase motor running at 1750 rev/min is used to drive a rotary pump, which operates 24 hours per day. An engineer has specified a 7.4-in small sheave, an 11-in large sheave, and three B112 belts. The service factor of 1.2 was augmented by 0.1 because of the continuous-duty requirement. Analyze the drive and estimate the belt life in passes and hours.

Solution

The peripheral speed V of the belt is

$$V = \pi dn/12 = \pi(7.4)1750/12 = 3390 \text{ ft/min}$$

Table 17–11: $L_p = L + L_c = 112 + 1.8 = 113.8$ in

Eq. (17–16b): $$C = 0.25\left\{\left[113.8 - \frac{\pi}{2}(11 + 7.4)\right] + \sqrt{\left[113.8 - \frac{\pi}{2}(11 + 7.4)\right]^2 - 2(11 - 7.4)^2}\right\}$$

$$= 42.4 \text{ in}$$

Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}(11 - 7.4)/[2(42.4)] = 3.057 \text{ rad}$$

$$\exp[0.5123(3.057)] = 4.788$$

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Example 17-4 continued

Interpolating in Table 17–12 for $V = 3390$ ft/min gives $H_{tab} = 4.693$ hp. The wrap angle in degrees is $3.057(180)/\pi = 175°$. From Table 17–13, $K_1 = 0.99$. From Table 17–14, $K_2 = 1.05$. Thus, from Eq. (17–17),

$$H_a = K_1 K_2 H_{tab} = 0.99(1.05)4.693 = 4.878 \text{ hp}$$

Eq. (17–19): $$H_d = H_{nom} K_s n_d = 10(1.2 + 0.1)(1) = 13 \text{ hp}$$

Eq. (17–20): $$N_b \geq H_d/H_a = 13/4.878 = 2.67 \rightarrow 3$$

From Table 17–16, $K_c = 0.965$. Thus, from Eq. (17–21),

$$F_c = 0.965(3390/1000)^2 = 11.1 \text{ lbf}$$

Eq. (17–22): $$\Delta F = \frac{63\,025(13)/3}{1750(7.4/2)} = 42.2 \text{ lbf}$$

Eq. (17–23): $$F_1 = 11.1 + \frac{42.2(4.788)}{4.788 - 1} = 64.4 \text{ lbf}$$

Solution of Problem 17-18

Two B85 V belts are used in a drive composed of a 5.4-in driving sheave, rotating at 1200 rev/min, and a 16-in driven sheave. Find the power capacity of the drive based on a service factor of 1.25, and find the center-to-center distance.

Solution

Given: two B85 V-belts with $d = 5.4$ in, $D = 16$ in, $n = 1200$ rev/min, and $K_s = 1.25$

Table 17-11: $L_p = 85 + 1.8 = 86.8$ in

Eq. (17-17*b*):

$$C = 0.25\left\{\left[86.8 - \frac{\pi}{2}(16 + 5.4)\right] + \sqrt{\left[86.8 - \frac{\pi}{2}(16 + 5.4)\right]^2 - 2(16 - 5.4)^2}\right\}$$

$= 26.05$ in *Ans.*

Eq. (17-1):

$$\theta_d = 180^\circ - 2\sin^{-1}\left[\frac{16 - 5.4}{2(26.05)}\right] = 156.5^\circ$$

From table 17-13 footnote:

$$K_1 = 0.143\,543 + 0.007\,468(156.5^\circ) - 0.000\,015\,052(156.5^\circ)^2 = 0.944$$

Table 17-14: $K_2 = 1$

Belt speed: $$V = \frac{\pi(5.4)(1200)}{12} = 1696 \text{ ft/min}$$

Use Table 17-12 to interpolate for H_{tab}.

$$H_{tab} = 1.59 + \left(\frac{2.62 - 1.59}{2000 - 1000}\right)(1696 - 1000) = 2.31 \text{ hp/belt}$$

$$H_a = K_1 K_2 H_{tab} = (1)(0.94)(2.31) = \boxed{2.18 \text{ hp}}$$

For a factor of safety of one,

$$n_{fs} = \frac{H_a N_b}{H_{nom} K_s} \Rightarrow \boxed{1} = \frac{\boxed{2.18}\boxed{(2)}}{H_{nom}\boxed{(1.25)}}$$

$$H_a = \frac{Hd}{Np}$$

$$H_{nom} = \frac{4.36}{1.25} = 3.49 \text{ hp}$$

$$\Delta F_a = \frac{63025 H_a}{n(d/2)}$$

$$T_a = \frac{\Delta F_a d}{2},$$

F_c — F_i — F_1 — F_2 — $n_{fs} = \frac{H_a N_b}{H_d}$

Timing Belts

A timing belt is made of a rubberized fabric coated with a nylon fabric, and has steel wire within to take the tension load. It has teeth that fit into grooves cut on the periphery of the pulleys (Fig. 17–15). A timing belt does not stretch appreciably or slip and consequently transmits power at a constant angular-velocity ratio. No initial tension is needed.

$F_i \approx 0$ mostly

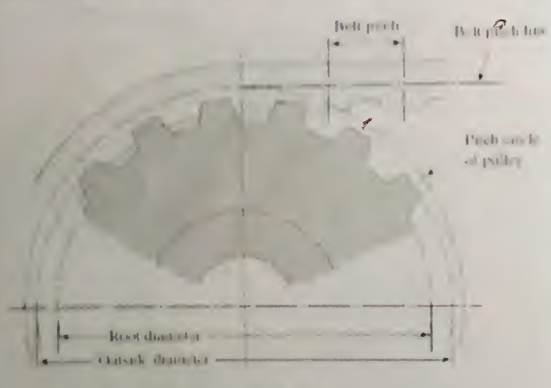


Figure 17–15

Table 17–18

Service	Designation	Pitch p, in
Extra light	XL	[illegible]
Light	L	[illegible]
Heavy	H	[illegible]
Extra heavy	XH	[illegible]
Double extra heavy	XXH	[illegible]

The five standard inch-series pitches available are listed in Table 17–18 with their letter designations. Standard pitch lengths are available in sizes from 6 to 180 in. Pulleys come in sizes from 0.60 in pitch diameter up to 35.8 in and with groove numbers from 10 to 120.

The design and selection process for timing belts is so similar to that for V belts

* Basic features of chain drives include: constant ratio, since no slippage or creep is involved; long life; and the ability to drive a number of shafts from a single source of power.

* Roller chains have been standardized as to sizes by the ANSI. Figure 17–16 shows the nomenclature.

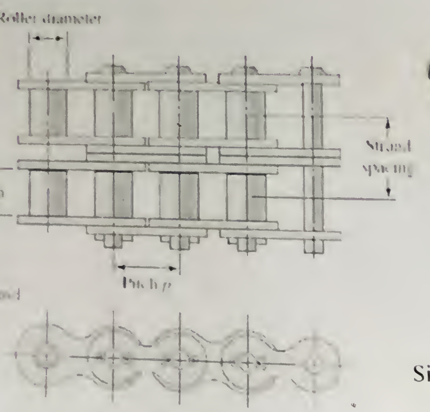
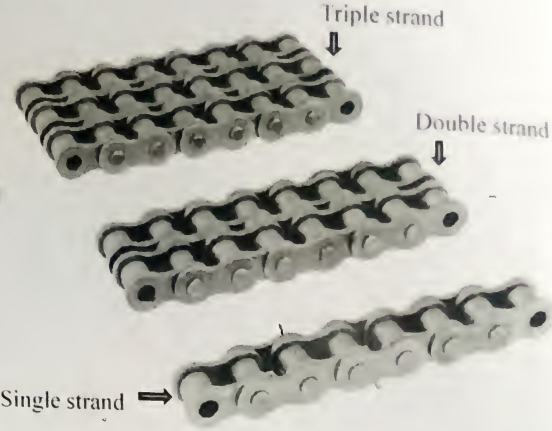


Figure 17-16



* These chains are manufactured in single, double, triple, and quadruple strands. The dimensions of standard sizes are listed in Table 17–19.

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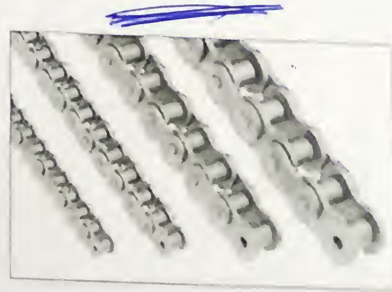
Roller Chains continued

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter, in (mm)	Multiple-Strand Spacing, in (mm)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)	0.130 (3.30)	0.252 (6.40)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)	0.200 (5.08)	0.399 (10.13)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)	0.306 (7.77)	—
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)	0.312 (7.92)	0.566 (14.38)
50	0.625 (15.88)	0.375 (9.52)	4 880 (21 700)	0.69 (10.1)	0.400 (10.16)	0.713 (18.11)
60	0.750 (19.05)	0.500 (12.7)	7 030 (31 300)	1.00 (14.6)	0.469 (11.91)	0.897 (22.78)
80	1.000 (25.40)	0.625 (15.88)	12 500 (55 600)	1.71 (25.0)	0.625 (15.87)	1.153 (29.29)
100	1.250 (31.75)	0.750 (19.05)	19 500 (86 700)	2.58 (37.7)	0.750 (19.05)	1.409 (35.76)
120	1.500 (38.10)	1.000 (25.40)	28 000 (124 500)	3.87 (56.5)	0.875 (22.22)	1.789 (45.44)
140	1.750 (44.45)	1.000 (25.40)	38 000 (169 000)	4.95 (72.2)	1.000 (25.40)	1.924 (48.87)
160	2.000 (50.80)	1.250 (31.75)	50 000 (222 000)	6.61 (96.5)	1.125 (28.57)	2.305 (58.55)
180	2.250 (57.15)	1.406 (35.71)	63 000 (280 000)	9.06 (132.2)	1.406 (35.71)	2.592 (65.84)
200	2.500 (63.50)	1.500 (38.10)	78 000 (347 000)	10.96 (159.9)	1.562 (39.67)	2.817 (71.55)
240	3.00 (76.70)	1.875 (47.63)	112 000 (498 000)	16.4 (239)	1.875 (47.62)	3.458 (87.83)

Table 17–19

Dimensions of American Standard Roller Chains—Single Strand



Assembly of single strand